Digital quantum memories with symmetric pulses

Q. Y. He, M. D. Reid, and P. D. Drummond *

Centre for Atom Optics and Ultra-fast Spectroscopy, Swinburne University, Melbourne VIC 3122, Australia
* pdrummond@swin.edu.au

Abstract: We propose a digital approach to quantum memories using a single-mode oscillator-cavity model, in which the coupling is shaped in time to provide the optimum interface to a time-symmetric input pulse. Our generic model is applicable to any linear storage medium ranging from a superconducting device to an atomic medium.

OCIS codes: (270.0270) Quantum optics; (270.5585) Quantum information and processing.

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1. Introduction

Quantum memories (QM) are key devices both for quantum information and fundamental tests of quantum mechanics. A QM will write, store then retrieve a quantum state after an arbitrary length of time. QM devices are considered vital for the implementation of quantum networks, quantum cryptography and quantum computing. At a more fundamental level, they could enable one to generate an entangled quantum state in one device, then test it’s decoherence properties in a different location. This would allow one to test the equivalence of the quantum state description for more than one physical environment. For example, there are proposals that gravitational decoherence may occur beyond the standard model of quantum measurement theory [1]. This would be testable with controlled ways to input, store, and then read out pairs of entangled quantum states in differing environments, with different separations or masses.

Here we propose a QM protocol, in which the quantum state is stored *digitally* in a dynamically switched cavity-oscillator system. In this approach one mode is input, one pulse at a time, allowing a *single* qubit or multilevel quantum digit (qudit) to be stored then retrieved. We derive a condition on the time-dependence of the oscillator-cavity coupling required to match to any external pulse-shape, including time-symmetric pulses. This contrasts with our previous work, in which the coupling was a step function, resulting in non-symmetric pulses having different shapes on input and output [2]. Because a QM uses time reversal as the key to retrieval, the use of time-symmetric pulses is essential to cascaded logic, where a retrieved output state must be re-used as an input state to other logic devices.

Digital memory protocols also contrast with proposals and experiments [3-8], in which an essentially analog QM is used to store any time-varying input, like an analog tape-recorder. Our digital QM proposal is highly suitable for single qudit quantum information processing, just as digital electronic memories are used in electronic computers. An essential feature of our treatment is that we show how a smooth, time-symmetric sech-pulse can be stored for times longer than the pulse duration, and recalled with high quantum fidelity. Thus, the output pulse shape replicates the input pulse. Hence, this type of quantum logic can be cascaded, with interchangeable inputs and outputs. Our analysis does not involve a slowly varying envelope approximation [9-13]. This allows the use of pulses that can be stored for times much longer than the pulse duration. We also analyze the effects of losses and noise in the coupling constant.

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The benchmarks for a QM are storage time and input-output fidelity. The memory time T must be longer than the duration T_I of the input signal: $T > T_I$. Otherwise, the memory is more like a phase-shifter than a memory. The final quantum state must also be a close replica of the original. In quantitative terms, the mean state overlap [14] between the intended and achieved quantum states (the mean fidelity $\bar{F}$) must satisfy $\bar{F} > \bar{F}_C$. Here $\bar{F}_C$ is the best mean fidelity obtainable with a classical measure and regenerate strategy [15]. Further to this, an ideal QM protocol must enable numerous sequential quantum logic operations to be performed, meaning many input-output "quantum states", carried on ingoing and outgoing pulse waveforms. This means that the output pulse envelope should be identical to that of the input.

Our theoretical calculations are carried out with linear oscillator models that are analytically soluble. This allows us to calculate pulse shapes that are dynamically matched in time to the cavity-oscillator system. This strategy can be combined with a variety of other technologies. This opens exciting experimental possibilities, including comparisons of fidelity in QM devices with different effective masses, as a fundamental test of decoherence in quantum mechanics.

Previous QM experiments were frequently limited by relatively short storage times [16]. Other demonstrations focus on retrieval efficiency at very high photon number [17]. However, these usually have a very low fidelity, since the fidelity at a fixed efficiency decreases exponentially with photon number. As a rule, previous proposals either ignore fidelity, or use criteria only applicable to special known states, like coherent or squeezed states [16,18,19]. It is more useful to allow for arbitrary input states. Our analysis is not restricted to any class of states, except for an upper bound on the input photon number.

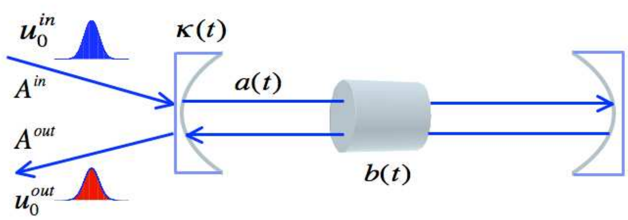

Fig. 1. Proposed dynamical atom-cavity QM. The cavity couples to only one ingoing and outgoing mode, $u_0^{in(out)}(t)$, and it is the quantum state of this mode that is stored. The pulse shape is optimized for efficient writing and reading of the state onto and from the oscillator medium inside the cavity. A symmetric pulse shape is used, so the time-reversed output is identical to the input.

2. Model

The quantum information in a temporal mode of the propagating single transverse-mode operator field $\hat{A}^{in}(t)$ is first transferred to an internal cavity mode with operator $\hat{a}(t)$, and then written into the oscillator or memory with mode operator $\hat{b}(t)$ up to time $t = 0$. Subsequently, the interaction is turned off or detuned for a controllable storage time T. The interaction is switched on again after time T, allowing readout into an outgoing quantum field $\hat{A}^{out}(t)$ at $t > T$ (Fig. 1). We treat quantum information encoded into single propagating modes that are temporally and spatially mode-matched to the memory device [20,21]. Here the relevant input and output mode operators are $\hat{a}_0^{in(out)} = \int u_0^{in(out)*}(t)\hat{A}^{in(out)}(t)dt$, where $u_0^{in(out)}(t)$ is understood to represent the input (output) temporal mode shape. For single-mode operation, the longitudinal mode spacing limits the bandwidth, so micron-scale cavities can achieve a THz bandwidth, while memory lifetime is only limited by the internal decay rate of the oscillator. We use the positive P-representation [22], in which all operators $\hat{A}^{in(out)}, \hat{a}_0^{in(out)}, \hat{a}, \hat{b}$ are formally replaced by c-numbers $A^{in(out)}, a_0^{in(out)}, a, b$. Using input-output theory [23], the dynamical equations are:

$$\begin{aligned}\dot{a}(t) &= -(\kappa + i\delta(t))a(t) + g(t)b(t) + \sqrt{2\kappa}A^{in}(t),\\ \dot{b}(t) &= -(\gamma + i\Delta(t))b(t) - g(t)a(t) + \sqrt{2\gamma}B^{in}(t).\end{aligned} \tag{1}$$

Here κ is the cavity damping, with detuning $\delta(t)$. The internal cavity-oscillator coupling is $g(t)$ (assumed variable), while the amplitude damping and detuning of the oscillator are $\gamma, \Delta(t)$ respectively, with an oscillator noise term B^{in} in the case of a finite temperature reservoir.

These equations can be applied to a range of experiments ranging from cold atoms to nano-mechanical oscillators. Depending on the system, the oscillator-mode interaction could involve quantum non demolition (QND) [24-26], Raman or electromagnetically induced transparency (EIT) [3-5], controlled reversible inhomogeneous broadening (CRIB) [6-8,27], superconducting transmission lines and superconducting quantum interference devices (SQUID) [28-30], magnetic control with a two-level atom, nano-mechanical oscillator storage [31,32] or even intra-cavity Bose-Einstein condensation (BEC) devices [33].

With Raman coupling, $g(t)$ represents a second Raman laser pulse shaped using an arbitrary waveform generator and an acousto-optic modulator. Cavity detunings $\delta(t)$ can be changed with piezo-electric actuators. Oscillator frequencies $\Delta(t)$ are tunable with external bias fields or techniques to change mechanical tension in nano-mechanical devices.

The completeness of the positive P-representation representation allows us to treat any input quantum state or memory protocol. Since the equations are linear, the overall time-delayed input-output relationship must be given by:

$$a_0^{out} = \sqrt{\eta_M}\, a_0^{in} + \sqrt{1-\eta_M}\, a^R. \tag{2}$$

Here an amplitude retrieval efficiency η_M is introduced for the time-delayed read-out, and a^R represents the overall effects of the loss reservoirs. For simplicity, all reservoirs are assumed here to be in the vacuum state. Hence, we can solve Eq. (1) to obtain $\sqrt{\eta_M} = a_0^{out} / a_0^{in}$ by integrating over the positive-P output field A^{out}. This is valid since a^R only acts on a zero-temperature reservoir, and is equal to zero in the positive P-representation.

We will analyze the mode-matching conditions for two different dynamical models with fixed cavity damping κ. We consider an input signal with a peak amplitude at $t = t_0$, and an output retrieved after a time-interval T, i.e., with a peak at $t = T + t_0$. In order to obtain dynamical mode matching we require an outgoing vacuum state for $t < t_0$. In the positive P-representation this translates to the simple requirement that $A^{out} = \sqrt{2\kappa}a - A^{in} = 0$, so that $A^{in} = \sqrt{2\kappa}a.$ The two models use strategies of either variable coupling or variable detuning to switch on and off the coupling between the oscillator and the intra-cavity field. For simplicity, we treat the case of zero internal damping ($\gamma T \ll 1$) in the equations, while still including oscillator damping in the graphs to demonstrate that this effect can be made small if necessary. With no loss of generality, we consider units for which $\kappa = 1$.

2.1 Variable coupling ($\delta, \Delta = 0$)

Here we propose that the cavity decay is fixed, and that $g(t)$, the interaction of the cavity field with the oscillator, is switched. During the input stage, the relation $A^{in} = \sqrt{2\kappa}a$ means that $a(t)$

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is given for any desired mode-shape $A^{in}(t)$. This gives an expression for $g(t)$, since from Eq. (1), with $\gamma \to 0$, one has $g(t) = -\dot{b}/a$. Hence:

$$[\dot{a} - gb]/a = \dot{a}/a + (\dot{b}^2)/(2a^2) = 1. \tag{3}$$

In order to realize a time-symmetric input mode with $a = a_0 \sec h(t - t_0)$, from Eq. (3) we see that the internal field amplitude must be $b = a_0 e^{(t-t_0)} \sec h(t - t_0)$. The optimal shape of the cavity-oscillator coupling in time is therefore $g(t) = -\dot{b}/a = -\sec h(t - t_0)$. This is independent of the amplitude a_0 which encodes the quantum information. Here the coupling is synchronized to t_0, which is the pulse arrival time. The quantum memory readout is obtained simply by time-reversal after half the memory storage time has elapsed, so that $g(t) = g(T + 2t_0 - t)$. The resulting output mode is also time-reversed and is unchanged apart from being inverted: $A^{out} = -\sqrt{2}a_0 \sec h(T + t_0 - t)$. A typical result is shown in Fig. 2(a), from integrating Eq. (1).

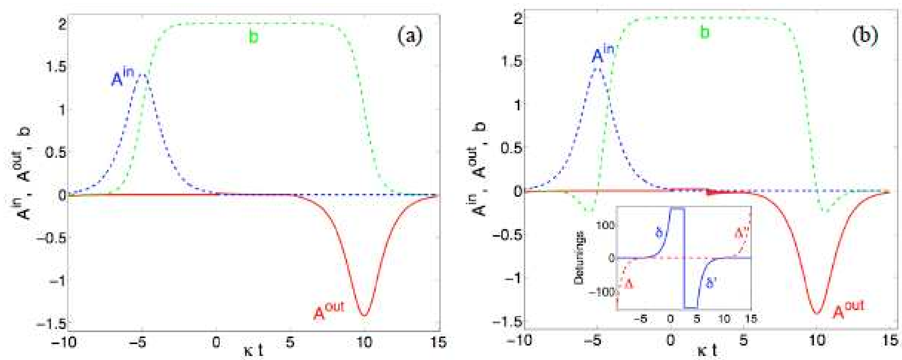

Fig. 2. (a) Case 1: Cavity input (dashed) and output (solid) amplitudes. The dotted line gives the oscillator amplitude. (b) Case 2: The inset gives the detuning shapes in time: $\Delta(t)$ and $\delta(t)$. Here $t_0 = -5$, $T = 15$, $a_0 = 1$.

To give an example, consider a trapped and cooled alkali-earth metal atomic vapor in a $100\mu m$ Fabry-Perot interferometer with a $25\mu m$ mode waist. Strontium has metastable ${}^3P_0, {}^3P_2$ states with decay rates of $\sim 6.1\times10^{-3}\,Hz$, or less, which are used for atomic clocks. There is a ${}^3S_1 \to {}^3P_0, {}^3P_2$ transition ($T_1 \sim .11\mu s$,) that is available for Raman switching, with no hyperfine structure if the abundant bosonic even isotope is used. The relevant signal and pump wavelengths are $679nm$ and $707nm$. We assume the cavity transmission is $T = 0.01$, giving a cavity decay lifetime of $\sim 67\,ps$. The effective coupling strength of ${}^3S_1 \to {}^3P_0$ is $g' \simeq Ng\Omega/\Delta \sim \kappa$ [10]. Here $g = g_0 |u(\vec{r})|$ is the coupling parameter [20], which includes the normalized mode function $u(\vec{r})$; $\Omega = \mu E/\hbar$ is the peak Rabi frequency of the Raman pump field E that couples the two levels ${}^3S_1 \to {}^3P_2$ with the atomic dipole element μ. According to the formula of [20], we need a maximum value of $g \sim 3.5\times10^7\,Hz$. Assume we have $N \sim 10^3$ trapped atoms, with a Raman detuning of $\Delta \sim 10\kappa$, then $\Omega \sim 2.1\kappa$. From the relation $T_1 = 3\hbar\varepsilon_0 c^3/(4\omega^3 |\mu^2|)$, we obtain $\mu \sim 0.29\times10^{-29}\,C\cdot m$ and therefore a peak intensity of $I \sim 400W/mm^2$, which is an achievable pulse intensity.

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2.2 Variable detuning

In this approach, the coupling is changed by varying the detunings $\Delta(t)$ and $\delta(t)$. We consider the simplest case with $g=\kappa=1$ and a symmetric pulse $a=a_0 \sec h(t-t_0)$. To give a vacuum output during the writing phase we must have: $\Delta = i(\dot{b}+a)/b$, $\delta = i(\dot{a}-a-b)/a$. We suppose that $b=b_1+ib_2$, so that $\delta = i(\dot{a}-a-b_1)/a + b_2/a$, $\Delta = [(\dot{b}_1+a)b_2 - \dot{b}_2 b_1]/|b|^2 + i[(\dot{b}_1+a)b_1 + \dot{b}_2 b_2]/|b|^2$. $\Delta = [(\dot{b}_1+a)b_2 - \dot{b}_2 b_1]/|b|^2 + i[(\dot{b}_1+a)b_1 + \dot{b}_2 b_2]/|b|^2$. Since $\mathrm{Im}(\Delta)=\mathrm{Im}(\delta)=0$, we find that $b_1 = (\dot{a}-a) = -a_0 e^{(t-t_0)} \sec h^2(t-t_0)$, and hence that $b_2 = a_0 e^{(t-t_0)} \sec h(t-t_0)\tanh(t-t_0)$. Finally, to realize symmetric input and output pulse shapes, we obtain the required detunings of: $\Delta = e^{-(t-t_0)}\tanh(t-t_0) + \sec h(t-t_0)$, $\delta = e^{(t-t_0)}\tanh(t-t_0)$.

After a controllable storage time, the interaction is switched back by time reversal of the detunings, so that $\delta' \to -\delta$ and $\Delta' \to -\Delta$, as shown in Fig. 2(b).

3. Loss and coupling noise

A useful quantum memory must be immune to loss and noise. We include a white-noise component in the coupling constant, so that $g(t) \to g(t) + n_0\zeta(t)$, to simulate experimental fluctuations. Here we set $\langle \zeta(t)\zeta(t') \rangle = \delta(t-t')$ to correspond to broadband noise. Figure 3 shows how quantum memory fidelity $\bar{F}$ in Case 1 during a storage time of $T=15$ is affected for a range of noise levels, which corresponds to various losses: $\gamma/\kappa = 0,\ 0.005,\ 0.01$. Here we have numerically solved Eq. (1) and integrated the mode overlap with the required sech mode function to obtain the value of $\sqrt{\eta_M}$ from Eq. (2), averaging over 100 different noise realizations. $\bar{F}$ is calculated based on our previous work [2] for coherent input states with mean photon number $\bar{n}=20$. Decreasing γ increases the storage efficiency even further.

With coupling noise of $n_0 = 0.05$ and a residual loss $\gamma/\kappa = 0.01$, we also calculate how the memory fidelity varies with different relative storage times of $15, 20, 25, 30$. For coherent input states having a mean photon number $\bar{n}=20$, we find the average fidelities $\bar{F} = 0.70, 0.58, 0.47, 0.39$. The first two fidelities are both above the classical bound $\bar{F}_{20}^{c} = 0.51$ required for a quantum memory. For arbitrary input states of up to two photons, the fidelities are $\bar{F} = 0.81, 0.76, 0.71, 0.67$, which are all above the classical bound of $\bar{F}_2^{b} = 0.5$. For these parameters, we are able to predict a quantum memory with both high fidelity and relatively long memory lifetime, even with internal loss and coupling parameter noise.

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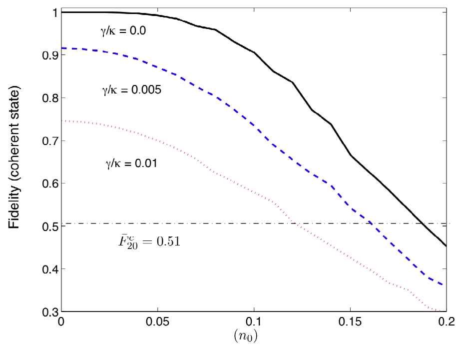

Fig. 3. The Fidelity (case 1 for coherent state with $\overline{n} = 20$) as a function of coupling noise with various loss ratios: $\gamma / \kappa = 0$ (solid), 0.005 (dashed) and 0.01 (dotted).

4. Conclusion

In conclusion, we propose a general protocol for a digital quantum memory, using a cavity-oscillator model. We show that with temporal modulation of coupling and/or detuning, it is possible to mode-match to identical time-symmetric input and output pulses. This type of quantum memory promises to give both high quantum fidelity and long lifetimes.

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